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United States  
Department of  
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Pacific Northwest  
Forest and Range  
Experiment Station

Research Paper  
PNW-345  
November 1985



SD11  
A45h  
NO. 345

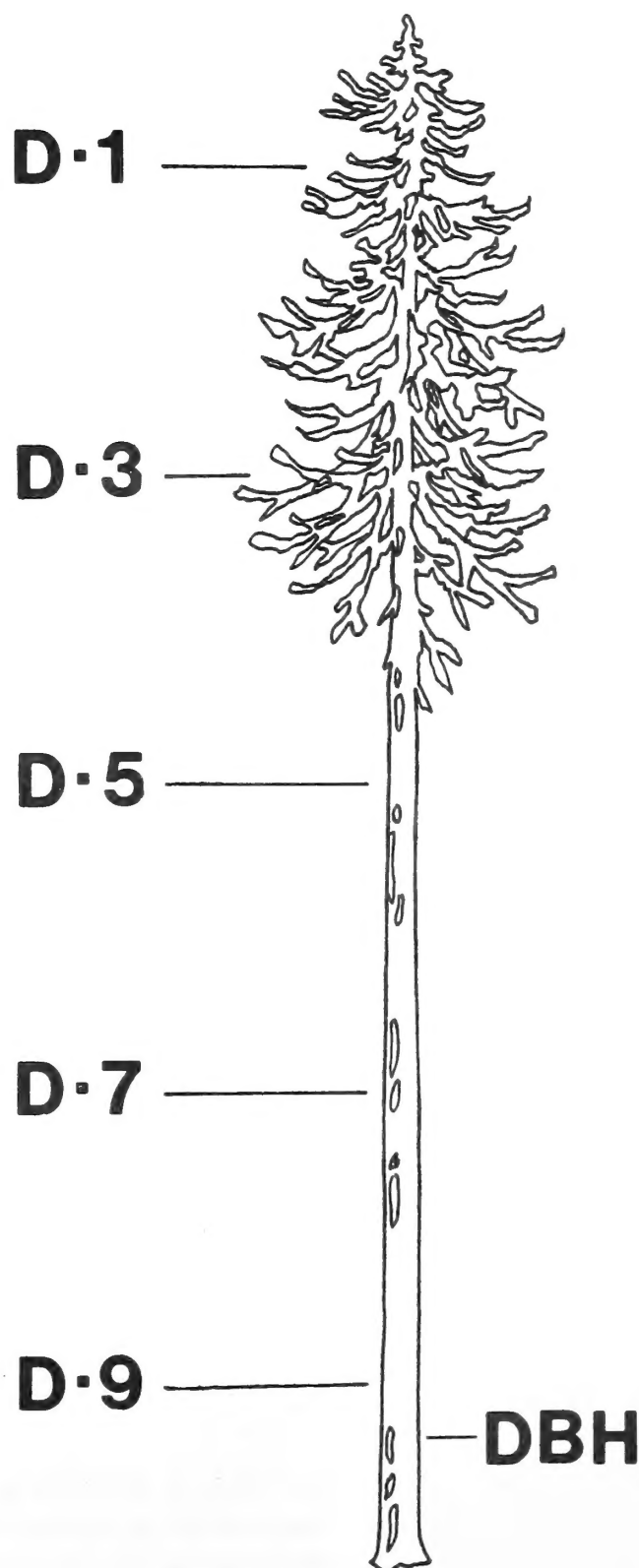
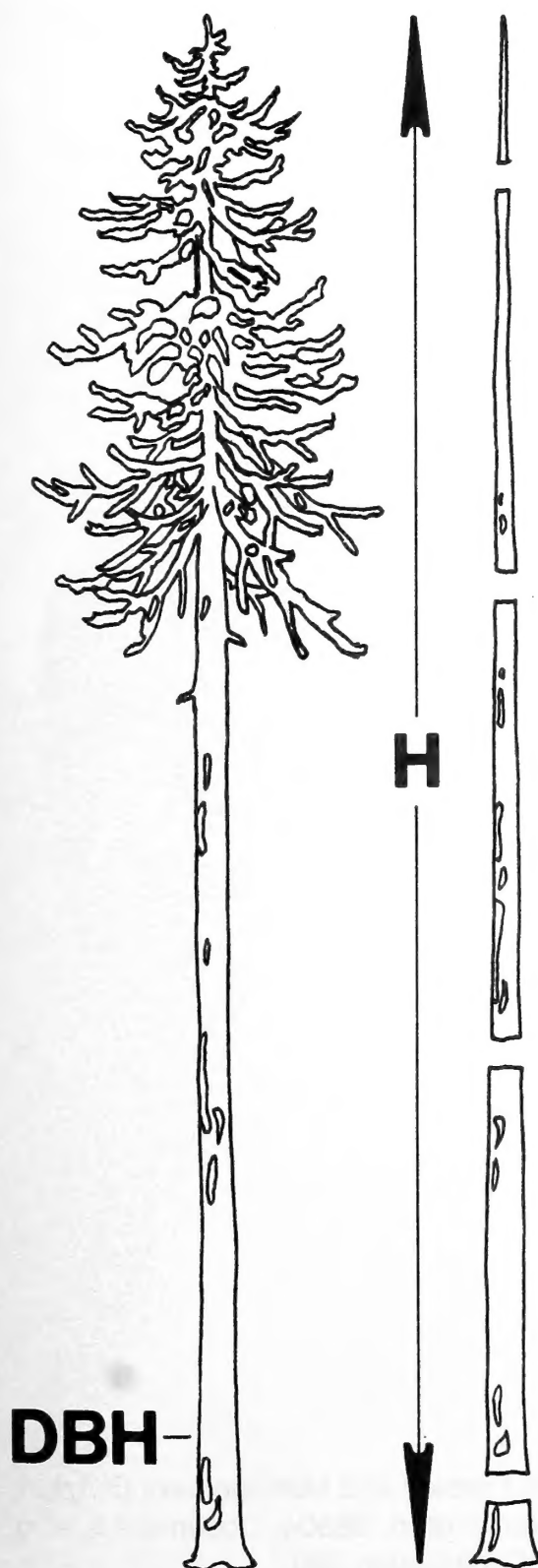
# Tree Form Quotients as Variables in Volume Estimation

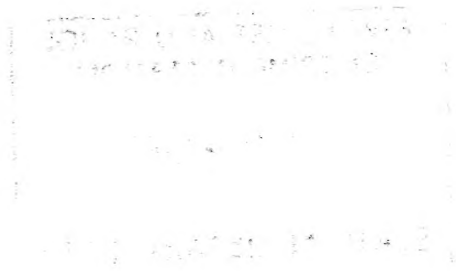
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## Abstract

**Hoyer, Gerald E.** Tree form quotients as variables in volume estimation. Res. Pap. PNW-345. Portland, OR: U.S. Department of Agriculture, Forest Service, Pacific Northwest Forest and Range Experiment Station; **1985**. 16 p.

The study reviews Hohenadl's procedure for defining form quotients and tree volume from diameters measured at fixed proportions of total tree height. Modifications of Hohenadl's procedure were applied to two sets of data for western hemlock (*Tsuga heterophylla* (Raf.) Sarg.) from the Pacific Northwest. The procedure was used to define volume differences in thinned stands, and selected form quotients were used as variables to improve accuracy and precision of standard tree volume equations. Estimating form quotients on standing trees requires less time than complete stem dendrometry. The technique has application to other tree species.

Keywords: Volume estimation, volume equations, form factors, form quotient, western hemlock.

## Summary

The study briefly reviews Hohenadl's procedure for defining form quotients and estimating tree volume using stem diameters at fixed proportions of total tree height. Hohenadl's procedure was modified to improve accuracy and the modified procedure was used to define volume differences in thinned western hemlock (*Tsuga heterophylla* (Raf.) Sarg.) stands. Selected form quotients were used as variables in tree volume equations.

Young western hemlock stands, 20 feet tall, were thinned to 4-, 9- and 22-foot spacings. Upper stem diameters were measured on standing trees four times during an 8-year period following thinning. A standard volume equation using only tree diameter at breast height and total height overestimated tree volume when compared with that measured using upper stem diameters.

The form quotients (D.5/D.9), (D.9/DBH), (D.7/D.9), and others were calculated for 638 western hemlock sample trees by interpolating for unknown upper stem diameters when measurements at D.5, D.7, and D.9 were missing. Form quotients contributed significantly to accuracy of equations for estimating known tree volume. The quotients identified tree form differences and led to more precise estimates of tree volume than did use of diameter at breast height and tree height alone.

Estimating the form quotients required less time than complete stem dendrometry. The technique described could be used with other species for which stem measurement data are available and for which the specific form quotient measurements may not have been recorded.

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## **Introduction**

### **Review of Tree Volume Equations**

Standard volume tables (equations) are often used to estimate tree volume as a function of tree diameter and height for both routine forest measurement and for forest research purposes. A recognized shortcoming is that a standard volume table (equation) may fail to estimate the volume of sample trees in a specific stand (Evert 1968, Grosenbaugh 1954, Hazard and Berger 1972). This may happen if the actual taper of the sample trees of a stand differs from the average taper of the trees used in construction of the volume equation. Use of these volume equations ignores the variation that occurs because of taper (that is, form) differences.

One solution is to estimate tree volumes with a standard volume equation, then directly measure the volume of a sample of the population of interest using intensive dendrometry. A regression fitted to the data corrects the estimate made by the standard equation. The final result can be thought of as an adjusted local volume table (equation). A complex volume estimation procedure such as three-P sampling (Grosenbaugh 1965) is an efficient specialized parallel to this general procedure. Another parallel uses the results of intensive dendrometry or felled sample tree measurements as access to the tarif volume system (Turnbull and others 1963). The average stand tarif so derived defines the correct local volume equation.

One way to simplify the volume estimating procedure and at the same time improve accuracy of tree volume estimates is to make the standard volume equation sensitive to variation of stem form.

## Hohenadl's Technique

The German mensurationist Hohenadl developed a technique for estimating tree volume from diameters measured at proportional distances along the tree bole (Altherr 1960, Assmann 1970, Heger 1965). The measurements lead to form quotients, which describe tree form and also estimate tree volume with a minimum of effort. The technique divides the total tree bole into segments of five lengths, each one-fifth of total height. Diameters measured at midpoints of the five segments are referred to as D.1, D.3, D.5, D.7, and D.9. D.5 is at half the total tree height and D.9 is at 90 percent of the height from the tree tip to the ground. From these stem diameters Hohenadl develops form quotients and tree volume. The rationale of Hohenadl's method is given in the appendix.

Altherr (1960) and Assmann (1970) identify an approximate 4-percent underestimate of tree volume as calculated by Hohenadl's method. Five measurements failed to properly account for flare near the tree base. The underestimate can be corrected by using additional diameters measured at other proportions of tree height near the base of the tree. With nine diameters, tree volume can be estimated within 1 percent of volume estimates that use measurements every 6.6 feet along the total tree stem (Altherr 1960). The rationale of Hohenadl's method holds when additional proportional diameters are used to increase volume precision.

The important point is that several of the form quotients defined by Hohenadl's method are highly correlated with volume (Assman 1970, Heger 1965, Pollanshütz 1966). The form quotients are simple dimensionless ratios of the diameters at selected proportional heights.

It follows logically that if diameters are estimated at several of Hohenadl's proportional heights in an existing body of tree measurements, quotients that characterize tree form can be defined and might allow the existing data to be more useful.

## Objectives

There were two objectives for this study:

1. To determine if there is a significant change of lower bole form quotient and volume as the result of initial spacing treatment of young western hemlock (*Tsuga heterophylla* [Raf.] Sarg.).
2. To produce and evaluate volume equations that allow lower bole form quotients to be used along with height and diameter at breast height (DBH) for computing tree volume of western hemlock.

## Data Bases

Separate sets of data were used in the two portions of the study. The first set of data was for a spacing study in which stem measurements were made on standing sample trees. The measurements were (1) total tree height, in feet; (2) Hohenadl's (Heger 1965) five diameters at fixed proportions of total tree height, plus (3) four additional diameters as proposed by Altherr (1960). The points of bole diameter measurement, expressed as fractions of the distance from the tree tip down, were: D.1, D.3, D.5, D.7, D.82, D.86, D.9, D.94, and D.98 as well as DBH (4.5 feet above ground). Diameters were measured both outside and inside bark.

The second set of data was for trees collected for volume table construction. Data for 638 sample trees were collected by the Pacific Northwest Forest and Range Experiment Station, USDA Forest Service; the Weyerhaeuser Company; and the State of Washington, Department of Natural Resources. The data were pooled to improve the standard volume equation for western hemlock, and results have been reported (Chambers and Foltz 1979, Wiley and others 1978). The trees had been measured for diameter outside and inside bark beginning at the stump and at 8- to 20-foot intervals thereafter to the tree tip. Tree DBH and total height were also measured.

## Study of Form Quotient Change

### Methods

**Sample tree description.**—Initial spacing treatments might cause form quotient and volume differences that would be undetected by tree height and diameter measurements alone. By using special instruments and climbing standing trees, nine bole measurements were made periodically for 4 years of an 8-year period on each of nine trees on two plots in three spacings. Height and DBH were measured annually for the 8 years. The three spacings were 4, 9, and 22 feet. All plots had been spaced to about 4 feet 2 years prior to treatment. Trees averaged 20 feet tall at first treatment and grew to about 45 feet after 8 years.

**The use of mean tariff number for volume calculations.**—The sample trees are the basis for assigning volume to all trees in a plot at each remeasurement. The procedure for assigning plot volume is the tariff system (Turnbull and others 1963), which uses the recognized linear relationship of tree volume to basal area in even-aged stands (Hummel 1955). From the assortment of volume lines provided by the tariff system, the appropriate volume-basal-area line is selected for each sample tree at each remeasurement. (Established mathematical relationships describe the selection process. Either direct tree measurements or height-diameter-volume equations may be used to estimate tree volume for use in the process.) Each line so selected has a unique, representative tariff number. The average tariff number for all the sample trees on a plot identifies the average volume-basal-area line for each plot.

Mean tariff number at each plot remeasurement is a direct index to the volume line. In this study the tariff procedure was used to compare two sources of the volume line; one was assigned from a standard volume equation and the other was calculated from stem measurements. Expressing the volume lines in terms of mean tariff number allows direct comparison of the sources of volume estimates. Significant differences of mean tariff number between the two sources imply that there is a difference in the underlying relationship of volume to basal area for the sources.

The mean treatment tariff number was calculated for each spacing at each remeasurement and trends were smoothed over time. Smoothing required estimation of the likely trends for years for which upper stem measurements were missing. The missing points are obvious on figure 1. The smoothed lines were drawn proportional to the smoothed lines derived from the annual measurements assigned from the standard volume equation.

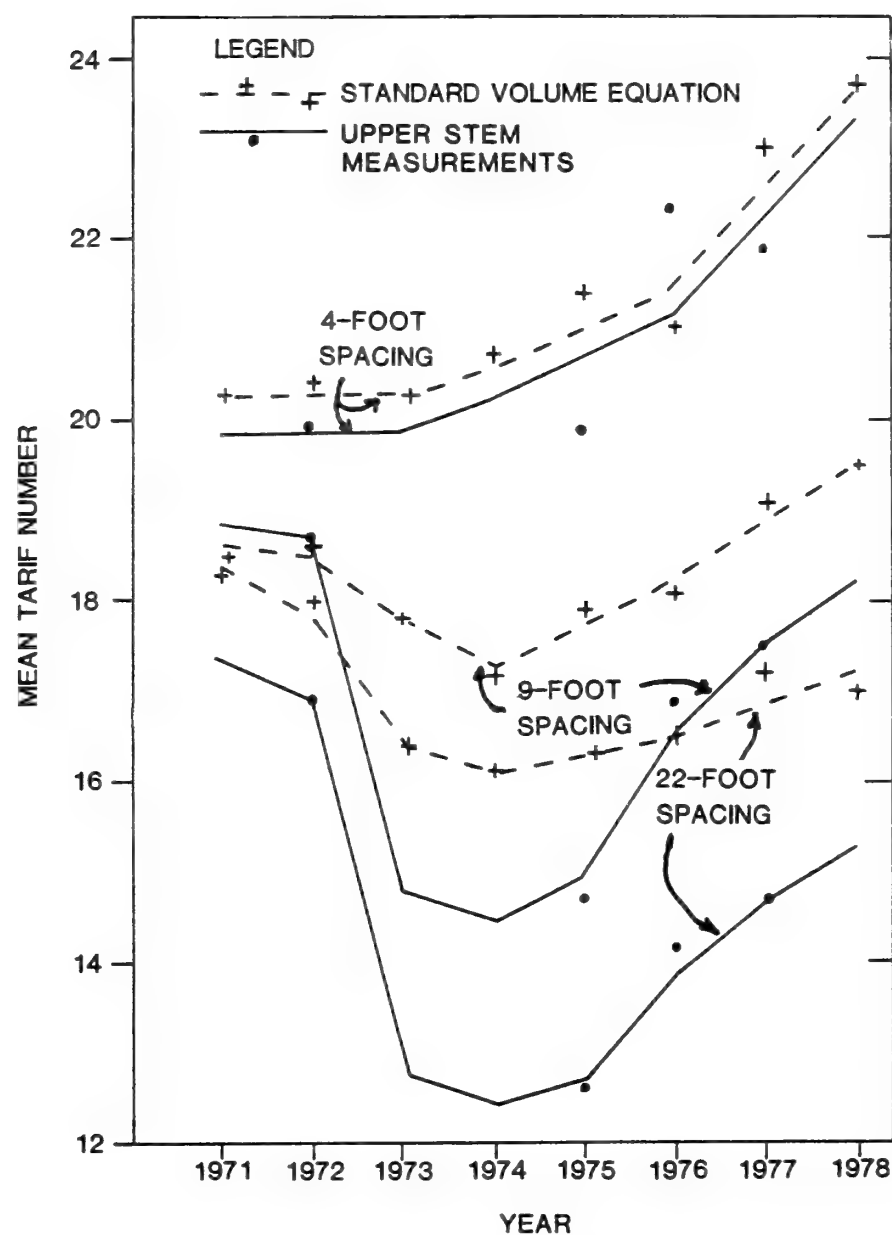


Figure 1.—Mean tariff number for early spacing treatments and two methods of volume estimation.

## Results

Differences illustrated in figure 1 resulted from differences in method of volume estimation. Volumes were lower when using upper stem measurements than when using the standard volume equation (equation 10 in tables 1 and 2) applied to height and diameter of the plot sample trees. The general trends of the two sets of tariff numbers were similar, but the rate of change was more abrupt when using upper stem measurements. The reduction followed by an increase of tariff number shown for the 9- and 22-foot spacings in year 1973 and later (fig. 1) is typical of the trend of tariff number following thinning. As tree diameter increased in response to thinning, subsequent tariff numbers became less than that of nonthinned trees of the same height.

**Table 1—Equation variables,  $R^2$ , and standard error of estimate for computing the logarithm of total cubic-foot volume, western hemlock trees**

Variable	Equation number											
	1	2	3	4	5	6	7	8	9	10	11	12
Intercept	X	X	X	X	X	X	X	X	X	X	X	X
$\log (DBH^2 \times H)$	X	X	X	X	X							
$\log (D.7/DBH)$		X	X	X	X							
$(D.9/DBH)^2$			X	X	X							
$(D.5/D.9)^2$				X								
DBH					X					X	X	X
$\log (D.9^2 \times H)$						X	X	X	X			
$(D.5/D.9)^{0.5}$							X	X				
D.7/D.9								X	X			
$\log DBH$										X	X	X
$(D.7/DBH) \times H$									X			
$\log H$									X	X	X	X
$\log (D.9/DBH)$											X	X
$\log (D.5/D.9)^2$												X
Adjusted $R^2$	0.9968	0.9994	0.9995	0.9997	0.9996	0.9976	0.9996	0.9997	0.9946	0.9978	0.9987	0.9997
Standard error of estimate												
log form	.0514	.0219	.0205	.0157	.0187	.0445	.0175	.0158	.0189	.0427	.0322	.0167
antilog form	1.12	1.05	1.05	1.04	1.04	1.11	1.04	1.04	1.04	1.10	1.08	1.04

$\log$  = logarithm to base 10;

H = total tree height above ground, in feet;

DBH = diameter outside bark at 4.5 feet above ground; and

D.5, D.7, D.9 = diameter outside bark at 0.5, 0.7, and 0.9 of total tree height from the tip down.

**Table 2—The most useful tree volume equations with coefficients**

Equation number	Equation coefficient	Variable
5	$\log \text{CVTS} = -2.73456265$	
	$+ 1.03313233$	$\log (\text{DBH}^2 \times H)$
	$+ .10407758$	$(\text{D}.9/\text{DBH})^2$
	$+ 1.32901244$	$\log (\text{D}.7/\text{DBH})$
	$-.00221155$	DBH
7	$\log \text{CVTS} = -3.37857130$	
	$+ 1.00767318$	$\log (\text{D}.9^2 \times H)$
	$+ .89111553$	$(\text{D}.5/\text{D}.9)^{0.5}$
8	$\log \text{CVTS} = -3.40978986$	
	$+ 1.00915622$	$\log (\text{D}.9^2 \times H)$
	$+ .59387586$	$(\text{D}.5/\text{D}.9)^{0.5}$
	$+ .31320765$	$(\text{D}.7/\text{D}.9)$
10	$\log \text{CVTS} = -2.71907159$	
	$+ 2.02477817$	$\log \text{DBH}$
	$-.00590929$	DBH
	$+ 1.07716464$	$\log H$
12	$\log \text{CVTS} = -2.53283870$	
	$+ 2.03621703$	$\log \text{DBH}$
	$-.00140209$	DBH
	$+ 1.01277022$	$\log H$
	$+ 1.76284601$	$\log (\text{D}.9/\text{DBH})$
	$+ .36689238$	$\log (\text{D}.5/\text{D}.9)^2$

CVTS = total tree cubic foot volume inside bark;

$\log$  = logarithm to base 10;

H = total tree height above ground, in feet;

DBH = diameter outside bark at 4.5 feet above ground; and

D.5, D.7, D.9 = diameter outside bark at 0.5, 0.7, and 0.9 of total tree height from the tip down.

**Change of tariff number with treatment and time.**—Estimates of mean tariff number from stem measurements and the standard volume equation were nearly the same in 1972. Average differences were 0.5, 0.1, and 1.1 tariff units for the three treatments and were statistically nonsignificant. Mean tariff numbers became statistically significant (at the 5-percent confidence level) with time and developed different trends (fig. 1). The two techniques produced significantly different volume relationships for treatments.

**Influence of volume estimation method on volume.**—The standard volume equation overestimated volume derived from upper stem measurements by an amount that was in direct proportion to the difference in tariff number. By 1978 the standard volume equation overestimated the 9-foot spacing by 7 percent and the 22-foot spacing by 12 percent.

## Conclusions

The standard volume equation, by its insensitivity to differences in tree form, significantly overestimated volume in these young stands.

It is uncertain what the future course of treatment differences will be; therefore, future measurements should allow evaluation of tree form. Dendrometer measurements along the full bole of well-crowned western hemlock trees will be difficult. A method is needed that includes form quotients in the volume estimating process. The next section describes development of such a method.

## Measured Form as a Third Variable in Diameter-Height Volume Equations

Volume equations, which depend just on height and diameter, account for tree form differences only to the extent that form is predictable from height and diameter. Volume depends on a combination of diameter, height, and form, yet only rarely is form included as an additional variable in standard volume equations. In the Pacific Northwest, tree volume tables often use a fixed point on the upper bole, usually 16 or 32 feet above ground, as a basis for a form measurement.

## Background

Heger (1965) in his application of Hohenadl's method reemphasizes that tree shape can be characterized using form quotients for diameters selected at proportional points of reference on the tree bole. As shown in the appendix,  $(D.5/D.9)$  is a form quotient that represents shape of the tree and is highly correlated with tree volume. Reukema (1971) uses  $D.5$  as a basis for form-volume estimation and Roebbelen and Smith (1981) find that a diameter measurement at half height improves the precision of volume estimation. To some extent, the lower bole form quotient,  $(D.9/DBH)$ , represents the influence of measuring diameters at proportional heights among short and tall trees.

Pollanschultz (1966) examines form functions and volume equations derived from them. He uses the variables  $DBH$ , total height, and stem diameters at "0.1, 0.3, and 0.5 of total height"—the equivalent of  $D.9$ ,  $D.7$ , and  $D.5$ , respectively, as defined here. He finds that combinations of these primary variables effectively reduce standard deviation of form-function equations. Schmid-Haas and Winzeler (1981) find that including upper stem diameter is very important for tree volume estimation. They use either diameter  $D.7$ , as defined here, or the diameter at a fixed height of 23 feet. They find that volume functions using only diameter at breast height provided volume estimates with standard deviations 30 to 110 percent higher than volume functions with form included when the same instruments were used for both. It is clear that including a form variable increases the precision of tree volume equations.

Based on these findings, it was expected that  $(D.5/D.9)$ ,  $(D.7/DBH)$ ,  $(D.7/D.9)$ ,  $(D.9/DBH)$ , or their transformations would provide direct estimates of tree form that would serve as variables together with diameter and height in a tree volume equation. These quotients are easily measured on standing trees with a Spiegel relaskop, provided the points are visible, and require less time to estimate than does complete stem dendrometry.

## Methods

The most recent effort (Chambers and Foltz 1979) to strengthen the standard volume tables for western hemlock combined existing tree data with additional data for felled large trees. In all, 638 trees with detailed stem measurements were available for analysis.

**Computing tree volume and interpolating for missing diameters.**—Cubic foot volume of each section was computed as that of a frustum of a neiloid or of a paraboloid for sections of the bole between the stump and the tip section. A plotted sample of 19 representative trees showed that the paraboloid was appropriate for tree sections occurring at distances up to 93 percent of the distance from the tree tip to the ground. The neiloid was appropriate for tree sections occurring in the remaining 7 percent of the distance. Exceptions to this were trees less than 45 feet tall, which were treated as neiloids from stump to DBH. The inflection point at 93 percent of the distance from the tip is lower than the 75 to 80 percent for all species that Demaerschalk and Kozak (1977) note.

Volume of the tip section was computed as that of a cone, and volume of the stump as that of a cylinder using stump diameter. The volume computed from each section was summed to total wood volume, inside bark.

Only a small part of the data base had direct measurements for  $D.5$ ,  $D.7$ , and  $D.9$ . These missing measurements were estimated by interpolating for diameters using both paraboloidal and neiloidal equations for appropriate cross section areas of each tree. Diameters that resulted from using these formulas were the same as those produced using a direct linear interpolation of section diameters because the tree sections were relatively short.

Grosenbaugh (1966) states that it matters little which formula is used for interpolation provided the distance between measured points is short. He recommends distances such that the one measured diameter is within 20 percent of the diameter of the other. A random sample of 25 trees in this study showed that only 7 percent of the three lower bole interpolations exceeded Grosenbaugh's recommendation; 5 of that 7 percent were measurements on short trees where the interpolation was between measurements only 4 feet apart.

The following form quotients were computed using outside bark measurements:  $(D.5/D.9)$ ,  $(D.7/D.9)$ ,  $(D.9/DBH)$ , and  $(D.7/DBH)$ . In addition, the logarithm, the square, and the square root of each form quotient were computed and each multiplied by tree height and included as variables in regression analysis. The variables and transformations selected by step-wise multiple regression appear in table 1.



**Testing results using the measured range of form quotient values.**—Original tree data were arrayed in order by values of height and diameter, listing  $\log (D.9/DBH)$ ,  $\log (D.7/DBH)$ , and  $\log (D.5/D.9)^2$ . From this array, maximum and minimum values of the three form quotients were plotted over height for similar sample trees having diameters within 0.8 inch and heights within 0.5 foot of each other.

The data had distinct trends, but because there were never more than two to four trees with similar diameter and height, the plotted points of maximum and minimum values for the form quotients were erratic. Trends of the average high and average low values for the appropriate form quotients were smoothed by hand and used to estimate volume in equation 12. (Equation 12 is directly comparable to equation 10, the form most commonly used in the Pacific Northwest.) These volumes are practical estimates of the influence of the form quotients on volume accuracy when diameter and height are held constant. The smoothed values are summarized in table 3.

**Table 3—Smoothed high and low values of 3 form quotients; sample tree diameter at breast height and total tree height are held constant**

DBH	Height	Form quotients					
		D.9/DBH		D.7/DBH		D.5/D.9	
<i>Inches</i>	<i>Feet</i>						
2.2	20	1.05	1.29	0.87	0.94	0.39	0.60
4.6	30	1.02	1.11	.83	.88	.52	.69
5.0	50	.99	1.00	.83	.88	.71	.79
10.0	70	.97	.99	.83	.88	.72	.77
10.3	100	.94	.97	.82	.88	.71	.76
15.0	110	.94	.97	.82	.88	.71	.76
17.0	120	.93	.96	.81	.88	.70	.75
18.0	130	.93	.96	.80	.87	.69	.74
18.0	140	.92	.96	.77	.86	.69	.73
33.0	160	.91	.96	.73	.85	.67	.71
25.0	180	.90	.96	.70	.83	.62	.65

The trees used earlier in the study were also examined to determine the range of form quotients as they occur in comparable stands subjected to early spacing treatment. The results are in table 4.

**Table 4—Range of values of individual tree form quotients, paired stands,<sup>1</sup> thinned to different early spacings**

Stand and treatment	Number of trees	DBH	Height	Form quotients		
				D.9/DBH	D.7/DBH	D.5/D.9
		<i>Inches</i>	<i>Feet</i>			
1979:						
4-foot spacing	10	3.6-5.8	35-46	1.00-1.08	0.85-0.98	0.70-0.83
22-foot spacing	10	6.9-10.1	36-45	1.00-1.03	.75-.84	.49-.59
1975:						
4-foot spacing	14	2.9-4.8	28-39	1.00-1.03	.79-.97	.51-.73
22-foot spacing	11	4.4-6.0	25-36	1.00-1.11	.60-.94	.45-.53

<sup>1</sup>Data from two ages of stands described in the first part of the study.

Twenty-three pairs of trees representing the total range of diameters and heights in the data base had nearly identical height and diameter. Volume differences of the paired trees ranged from 4 to 30 percent and averaged 11 percent. They were examined to see if the form of the lower bole (D.9/DBH) had greater influence on volume than did upper tree form (D.5/D.9). Of the 23 cases, (D.9/DBH) alone had greater influence on volume in 8 cases; (D.5/D.9) predominated in 3 cases and the two ratios shared influence in 10 cases. In 2 cases it was unclear which had more influence.

Useful equations were desired to supplement the main purposes of the study. The tariff volume-estimating procedure (Turnbull and others 1963) allows direct estimation of tree volume in a stand by averaging sample-tree tariff numbers. This avoids the need for fitting tree height-diameter curves. Estimates of tree height are often desired, however, for each tree on a plot. The data in this study allow derivation of equations for estimating tree height when volume and average tariff number are known.

Trees in the data base were assigned a tarif number based on actual tree volume according to the equations given by Brackett (1977). The data were then arrayed in ascending order of tarif number, and sequential groups of 20 trees were selected beginning with the tree of lowest tarif number. The two groups with the highest mean tarif number and the two with the lowest all had ranges of four to seven tarif number units within their respective group. The remaining groups had a range of about two tarif units. These ranges are not unusual for natural even-aged stands. Mean tarif was computed for each group of 20 trees and tree volume was estimated from mean tarif number. This estimated volume was used in equations for estimating tree height.

## Results and Discussion

**Tree volume equations.**—The 12 equations for predicting total cubic foot volume, and appropriate statistics, are in table 1. Selected equations complete with coefficients are in table 2. All variables included in each equation were significant at the 5 percent level of probability or higher.

Form quotients contributed significantly to increased precision of the cubic-foot volume estimate. Equations 2, 3, 4, and 5 added form quotients to the variable  $\log(\text{DBH}^2 \times H)$  used alone in equation 1. In a similar way, equations 7 and 8 added form quotients to  $\log(D.9^2 \times H)$  used in equation 6. Equations 11 and 12 added to  $\log(\text{DBH})$  and  $\log(H)$  used in equation 10. The contribution of each variable is visible in terms of increased value of  $R^2$  and in decreased value of the standard error of estimate in both logarithmic and antilog form, as shown in table 1.

Several form quotients including the variable D.5 were required to minimize the standard error of estimate (equation 4). Equations 5, 9, and 11 in table 1 exclude the variable D.5 and can be used on standing trees with relatively long crowns where D.5 is not visible. Equations 5 and 9 have lower standard errors of estimate than does equation 11. Equation 5 uses all the form variables as quotients, which makes it more convenient for field use than the variables in equation 9. For these reasons, equation 5 was considered as the most useful and was examined more thoroughly.

The residuals of equation 5 (in logarithmic form), when plotted with respect to its variables, had neither trends nor imbalanced variance. The residuals also had no bias when converted from the logarithmic form to actual values of each variable. Variance was not homogeneous, however. When trees were less than 45 feet tall, and D.9 was at or below the tree DBH, and the quotient  $(D.9/\text{DBH})$  was 1.00 or larger, there was less variance than when the quotient was less than 1.00. Also, most of the volume variance with respect to form quotient  $(D.5/D.9)$  occurred between quotient values of 0.6 to 0.8. For values above and below this range, there was less variance. Using the logarithmic form of the variables balanced the variance and maintained the requirements of regression analysis.

The value of the form variable related slightly to values of other independent variables; however, not to an extent that multicollinearity was considered to be a problem. Even with some multicollinearity, the equation parameters would provide unbiased estimates of volume.

The single form variable in equation 2 was stable with respect to tree diameter and height. Logarithm of volume was plotted over  $\log(\text{DBH}^2 \times H)$  for four equal divisions of the total range of value of the form variable  $\log(D.7/\text{DBH})$ . Generally, for each given level of  $\log(\text{DBH}^2 \times H)$ ,  $\log(D.7/\text{DBH})$  related clearly and consistently with volume. This illustrated the reliable behavior of the form variable within the range of the highly correlated relationship of tree volume to diameter and height.

Equation 10 was the equivalent of the equation given by Chambers and Foltz (1979); however, the coefficients were not identical. The coefficients differed because for this study the shape of the tree bole was treated as a neiloid up to one-tenth of tree height from the ground; Chambers and Foltz use the neiloid only as far as DBH. The difference between the two equations is trivial.

**The range of form quotient values and their effect on volume accuracy.**—Volume computed from tests of equation 12 shows that the influence of form has practical significance. The equation was solved using smoothed values for the average high and average low of the two form quotients while holding height and diameter constant.

Tree volume computed from the average high form quotients differed by about 9 percent from volume computed from the average lows for trees over 45 feet tall. Differences in volume in percent were 24, 40, 30, and 20 for trees 25, 30, 35, and 40 feet tall, respectively. Half the sample trees in the study had form quotient values outside the average highs and lows used in this test. If extreme values of form quotient are used, two trees over 45 feet tall with the same diameter and height could differ in cubic volume by as much as 18 percent. This difference, definable by using form quotients, would be unidentified when only tree diameter and height are known.

The smoothed high and low values of three form quotients (table 3) conservatively estimate form quotient variation when tree diameter and height are held constant. There was limited data to test the range of form quotient values on the treated stands described in the first part of this study. These ranges (table 4) are greater than those listed in table 3. The form values define differences among trees in widely and closely spaced stands. The values in table 4 in their respective equations translate into volume differences similar to those discussed in the first part of this study. In short, tree volume estimated by the equation was similar to volume that had been calculated originally from carefully measured tree stem diameters on trees not included among those used to generate the volume equations.

The examination of effects of  $(D.9/\text{DBH})$  and  $(D.5/D.9)$  on the volume of matched trees with identical DBH and height confirms a fact that can be deduced by considering the significant variables of the equations in table 1: That is, differences in form on the portion of a tree between DBH and D.9 and differences in form between D.9 and D.5 are both important in accounting for volume difference of trees with the same DBH and height.

**Useful supplemental equations**—Two equations given below estimate tree height when tree volume and average tariff are known. All variables in each equation had significant F-values at the 95-percent confidence level or higher. Plotted residuals for each variable showed neither bias nor imbalanced variance. Lowest standard error of estimate per given number of values was obtained when form quotients were included in the regression. An equation was derived for use when form quotients are unknown.

$$\begin{aligned}\log H = & 1.10955627 + 0.25628328 \log (\text{volume}) \\ & + 0.57283175 \log (\text{mean tariff}) - 0.78736802 \\ & \log (D.9/\text{DBH}) - 0.29643368 (D.7/\text{DBH})^2 \\ & - 0.07043974 (\text{DBH})^5;\end{aligned}$$

where:  $R^2 = 0.994$  and standard error of estimate,  $\log \text{form} = 0.0208$ .

$$\begin{aligned}\log H = & 0.70413539 + 0.24535889 \log (\text{volume}) \\ & - 0.01412038 (\text{volume})^5 + 0.59564993 \log (\text{mean tariff}) \\ & + 0.00010283 \text{DBH}^2\end{aligned}$$

where:  $R^2 = 0.982$  and standard error of estimate = 0.0372.

Mean tariff is the average of 20 trees. Volume in cubic feet for the total tree bole is estimated from mean tariff. Log is logarithm to base 10 and form quotients are as defined previously.

## Conclusions

As expected, using one or more form quotients increased the precision of volume computation beyond that given by standard volume equations that exclude a form variable. With height and diameter held constant, differences in stem form can lead to tree volume differences between 9 and 18 percent in trees over 45 feet tall. This is important, especially in cases where stand treatment can cause extreme differences in tree form.

Equation 8 is one of the two most precise of those given in table 2, but it requires estimates of D.9, D.7, and D.5 as well as height. Equation 5 is less precise than equation 8 but eliminates stem measurement at D.5, a point that is frequently not visible on standing trees. All form quotients are easily measured on standing trees with a Spiegel relaskop if the measurement points are visible from the ground.

Both equations 5 and 8 allow direct use of a form quotient when estimating tree volume. The added field measurement effort required is relatively small compared with taking multiple relaskop bole measurements up the whole tree stem length or felling trees for stem measurements. The equations adequately estimate the influence of stem form in treated western hemlock stands at a reasonable cost in time and effort. One needs to understand the influence of upper stem form to properly interpret results of spacing treatments. Such information is frequently lacking.

The procedure used here has other potential applications. Numerous collections of data on tree volume exist for various species but few, if any, have direct measurements of diameter at desired proportions of height. Because the interpolated estimates of these diameters proved to be effective in this study, similar success is likely if the method were applied to other tree species.

## Acknowledgments

Thanks are extended to Bruce W. Foltz and Jon Swanzy, State of Washington, Department of Natural Resources, for their help with the processing of the data. Thanks are also extended to the Weyerhaeuser Company for sharing data on felled trees, and to the Pacific Northwest Forest and Range Experiment Station, USDA Forest Service, who provided original data on felled trees from past work and funded collection of data for additional large-diameter trees.

## Metric Equivalents

The nominal height of 4.5 feet for measuring tree diameter is equivalent to 1.37 meters.

1 foot = 0.3048 meter

1 inch = 2.54 centimeters

1 cubic foot = 0.0283168 cubic meter

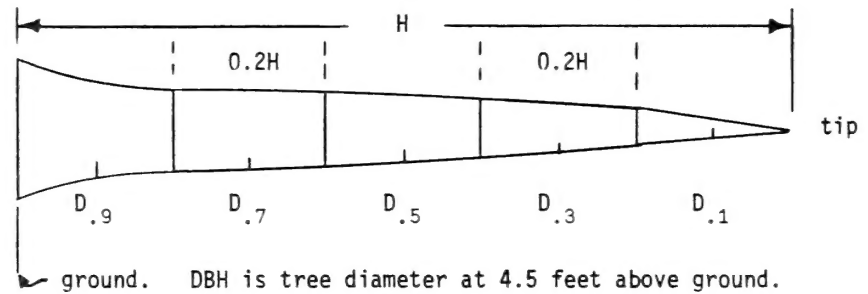
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## Appendix

### A. GIVEN: TREE DIMENSIONS WITH STEM MEASUREMENT AT 5 PROPORTIONAL POINTS:



### B. IT FOLLOWS THAT:

- LOG VOLUME FOR EACH OF THE FIVE SECTIONS USING SECTION MID-POINTS AND WITH DIAMETER AND LENGTH IN THE SAME UNITS OF MEASURE IS:

$$CV = \frac{\pi}{4} (0.2H) D_x^2$$

- TOTAL TREE VOLUME, CVTS, IS THE SUM OF THE VOLUME OF THE FIVE SECTIONS:

$$\begin{aligned} CVTS &= \frac{\pi}{4} (0.2H) \left[ D_{.9}^2 + D_{.7}^2 + D_{.5}^2 + D_{.3}^2 + D_{.1}^2 \right] \\ &= \frac{\pi}{4} (D_{.9}^2) 0.2H \left[ 1.00 + \left( \frac{D_{.7}}{D_{.9}} \right)^2 + \left( \frac{D_{.5}}{D_{.9}} \right)^2 + \left( \frac{D_{.3}}{D_{.9}} \right)^2 + \left( \frac{D_{.1}}{D_{.9}} \right)^2 \right] \end{aligned}$$

- THE RATIO OF TREE VOLUME TO VOLUME OF A CYLINDER OF DIAMETER  $D_{.9}$  AND LENGTH  $H$  IS CALLED  $LAMBDA_{.9}$  AND IS A REDUCTION FACTOR THAT ADJUSTS CYLINDER VOLUME TO VOLUME OF THE TREE SHAPE AND HENCE IS A "NATURAL" OR "TRUE" FACTOR.

$$LAMBDA_{.9} = \frac{CVTS}{\frac{\pi}{4} (D_{.9})^2 H} = 0.2 \left[ 1.00 + \left( \frac{D_{.7}}{D_{.9}} \right)^2 + \left( \frac{D_{.5}}{D_{.9}} \right)^2 + \left( \frac{D_{.3}}{D_{.9}} \right)^2 + \left( \frac{D_{.1}}{D_{.9}} \right)^2 \right]$$

- $LAMBDA_{.9}$  IS HIGHLY CORRELATED WITH  $\left( \frac{D_{.5}}{D_{.9}} \right)^2$

$$LAMBDA_{.9} = b \left( \frac{D_{.5}}{D_{.9}} \right)^2 + a$$

- $Fbh = CVTS / (DBH)^2 (H) =$  BREAST HEIGHT FORM FACTOR.

$$Fbh = LAMBDA_{.9} / (DBH / D_{.9})^2 = (D_{.9} / DBH)^2 (LAMBDA_{.9}) \text{ --- PRODAN, 1965, PAGE 50}$$

### C. THEREFORE:

- $(D_{.5} / D_{.9})^2$  REPRESENTS SHAPE OF STEM, THAT IS, THE "TRUE" FORM FACTOR.
- $(D_{.9} / DBH)^2$  REPRESENTS, AT LEAST IN PART, THE EFFECT OF DIFFERING PROPORTIONAL HEIGHTS OF MEASUREMENTS ON TREES OF DIFFERENT HEIGHT.
- $Fbh$  THE BREAST HEIGHT FORM FACTOR, IS A COMBINATION OF (1) AND (2).

THE SQUARE ROOT OF (1) IS IDENTIFIED AS A "TRUE" FORM FACTOR (HEGER, 1965) AND THE SQUARE ROOT OF (2), INVERTED, AS HOHENADL'S FORM QUOTIENT. (ASSMANN, 1970).



**Hoyer, Gerald E.** Tree form quotients as variables in volume estimation. Res. Pap. PNW-345. Portland, OR: U.S. Department of Agriculture, Forest Service, Pacific Northwest Forest and Range Experiment Station; **1985**. 16 p.

The study reviews Hohenadl's procedure for defining form quotients and tree volume from diameters measured at fixed proportions of total tree height. Modifications of Hohenadl's procedure were applied to two sets of data for western hemlock (*Tsuga heterophylla* (Raf.) Sarg.) from the Pacific Northwest. The procedure was used to define volume differences in thinned stands, and selected form quotients were used as variables to improve accuracy and precision of standard tree volume equations. Estimating form quotients on standing trees requires less time than complete stem dendrometry. The technique has application to other tree species.

Keywords: Volume estimation, volume equations, form factors, form quotient, western hemlock.

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